Applying and building on what we know: Issues at the intersection of research and practice

The Clements-Foyster Lecture

Dianne Siemon
RMIT University
<dianne.siemon@rmit.edu.au>

Identifying and building on what students know is widely regarded as essential to success in school mathematics. While this is difficult enough to achieve in practice, applying and building on what is known about mathematics learning and teaching seems to present an even greater hurdle. With growing inequity in who gets to choose and succeed in school mathematics, it is timely to consider why this might be the case. This presentation will consider some of the issues at the intersection of mathematics education research and practice from my perspective as a practitioner and researcher over four decades. It will conclude by suggesting that one way forward is for a greater, more collaborative focus on the relationship between curriculum, instruction, and assessment.

It is a great honour to be invited to present the Annual Clements-Foyster Lecture at MERGA 2019. The inaugural lecture was given by David Clarke in 2005 and it was preceded by a preamble written by Ken Clements that briefly documented the history of MERGA from its inception in 1977. A more extensive account of MERGA's evolution can be found in Clements (2012). As one of those who attended the very first meeting of MERGA at Monash University over 40 years ago, I am truly indebted to the great foresight of both Ken Clements and John Foyster in founding an organisation that conveyed an immediate sense of at-homeness, a shared sense of purpose, and where I was inspired and encouraged to pursue a career in mathematics education research and practice.

Fittingly, in relation to the theme of this years' conference, the theme of the 2005 conference was *Building Connections: Research, Theory, and Practice*. In his keynote, David made a case for critically examining the assumptions underpinning dichotomies such as "teacher-centred versus student-centred classrooms, real-world versus abstract tasks, and telling versus not-telling" (p. 15). His contention was that

> unless we can integrate each pair of categories as complementary elements of a more inclusive theoretical framework, we will remain unable to account for the diversity we find in international studies of classroom practice (p. 4)

My purpose in revisiting these dichotomies here is to point to what I see as the deification of 'research' implicit in these and other dichotomies such as reform versus traditional pedagogies, mixed-ability versus streamed classroom organisations, formative versus summative assessment, and even research versus practice. It is important to note the inverted commas here, my intention is not to position research above practice, but to draw attention to the dangers inherent in attempting to apply (implement/adapt) what is 'known' about teaching and learning mathematics to practice. When complex, highly-nuanced fields of inquiry are represented by metonymic often oppositional phrases such as 'reform versus traditional', the rich, qualified, and multifarious nature of what is known is compromised and when one approach/practice is privileged over the other on the basis of 'research' it invariably leads to a form of tribalism that pits individual against individual and group

2019. In G. Hine, S. Blackley, & A. Cooke (Eds.). Mathematics Education Research: Impacting Practice *(Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* pp. 49-67. Perth: MERGA.

against group in a way that is clearly unproductive[1]. While this seems to be an almost inevitable condition of Western society, this is not, as David Clarke (2005) pointed out, a necessary condition. Indeed, David resolved this issue, at least from the perspective of collaborative international classroom research, by suggesting that such dichotomies be seen as 'essential complementarities'.

The annual Clements-Foyster lecture is an opportunity to reflect on the conference theme through the lens of one's own research and what one holds dear. For example, in 2008, Helen Forgasz used three navigational metaphors to consider future directions of research on gender issues in mathematics education. In 2009, Bill Barton addressed the conference theme of *Crossing divides*, by a thoughtful and provocative consideration of mathematics knowledge for teaching from the perspective of a mathematician and mathematics educator. And, addressing the theme, *Mathematics: Traditions (and new) practices*, Rosemary Callingham (2011) suggested that it was time to "assess mathematics assessment and to reconsider the purpose, nature and use of assessment information" (p. 3). But the Clements-Foyster lecture is also an opportunity to step aside and consider the past, present, and future prospects of mathematics education and mathematics education research more generally (e.g., Clements, 2012; Gailbraith, 2014; Lowrie, 2015). It is my intention in what follows to both reflect on what I hold dear and to offer some thoughts on where to from here from my perspective as a practitioner and researcher over four decades. I will do that by considering three issues that I see at the intersection of research and practice: understanding change, the use (and abuse?) of 'data', and curriculum development. But first some background to set the scene.

Starting out

Two lessons learnt early in my first year as a secondary mathematics teacher served to shape the course of my professional journey. Asking about resources for my Year 7 class the day before school started, I was handed a textbook and told "we do the first three chapters in Term 1". Quite apart from the text's over-riding emphasis on procedures and practice, the content of the first three chapters; whole number arithmetic and some very elementary work with fractions and decimals; was to my mind the domain of primary school mathematics. Why were we doing this in Year 7? For some reason, there was no Faculty Head in place and the other Year 7 maths teachers (a science and physical education teacher by training) seemed happy enough to use the text. As a product of my generation, I decided in my absolute naivety (and with hindsight arrogance) not to use the text but to explore aspects of set theory using concrete materials and games. Towards the end of term, I was advised that the common test would be held the following week. There had been no mention of this previously and of course it was on the first three chapters of the book. On advising the students of the imminent test, I said that they were to give it their best shot but not to worry as we could focus on whatever they found difficult next term. When my Year 7 students did as well on the common test as the other Year 7 students, my views about the efficacy of using such texts to teach mathematics were confirmed. This set me on the path of exploring rich and engaging approaches to the teaching and learning of mathematics which I (and many others) subsequently pursued through our engagement with the *Study Group of Mathematics Learning* that Ken Clements had set up on arriving at Monash University in 1974.

The second lesson learnt as a result of this experience is best described as an epiphany. While the common test indicated that Year 7 students were reasonably confident with the

[1]Apparently, there is an evolutionary reason for 'black-and-white' thinking (Geher, 2016) even though on rational analysis we all know that the world is made up of many shades of grey – indeed, all the colours of the rainbow!

addition and subtraction of whole numbers, they were far less confident with problems involving multiplication, division, fractions, decimals, and percent. Of course, this would not surprise any experienced teacher of mathematics then or now, but it made me realise two things (a) that it was exceedingly unwise to make assumptions about what students know and (b) while I felt confident about teaching algebra, calculus and trigonometry, I had absolutely no idea how to address the number-related learning needs of my students. These realisations set me on the path of research in an unending quest to uncover and better understand what students know.

Understanding Change

While undertaking a Master of Education (by Research) degree in the early 1970s (on 4-year olds' capacity to order objects), I was extremely fortunate to have the opportunity to work as a maths method tutor in the Graduate Diploma (Secondary) program with Ken Clements, Charlie Lovitt, and Bernie Carter. These were heady years, secondary pre-service teachers were challenged to think way beyond texts, to explore abstract mathematics using concrete materials, and to focus on meaningful contexts. Building on the ideas of people like Zoltan Dienes, Caleb Gattegno, and Richard Skemp, the SGML was pushing the boundaries of what effective mathematics teaching might look like. Ken well remembers Charlie and Bernie accompanying him to weekend SGML conferences around Victoria at this time.

> The workshop leaders would sleep in tents in the local camping grounds, and we'd charge workshop participants $2 to attend the whole event. The workshops would start on Friday nights at 6 pm and go right through Saturday to about 10 pm. True! Those were the days! Always, the program was mostly made up of Dienes-type activities – there were very few lectures. (K. Clements, personal communication, 13 March 2019)

From the late 1970s and throughout the 1980s it was evident that these explorations into alternative ways of teaching and learning mathematics were beginning to bear fruit more generally. Innovative classroom problem-based resources such as *Reality in Mathematics Education* [RIME] (e.g., Lowe, 1980; Lowe & Lovitt, 1984) and the 'hands-on' *Mathematics Task Centre* materials (e.g., de Mestre & Duncan, 1980; Richards, 1985) were appearing in schools. Ground breaking professional development programs such as *Exploring Mathematics in Classrooms* [EMIC] (e.g., Beesey, 1989; Robinson, 1986) and the *Mathematics Curriculum and Teaching Program* [MCTP] (Lovitt & Clarke, 1986, 1988a, 1988b) were challenging prevailing classroom norms and supporting teachers to explore a broader range of pedagogical practices. Prompted in part by these developments and the *Agenda for Action* (National Council of Teachers of Mathematics [NCTM], 1980), problem solving became a major focus of professional conferences throughout the eighties (e.g., English, 1984; Horne, 1984; Siemon, 1985; Southwell, 1982) and led to a range of support materials for teachers (e.g., Stacey & Groves, 1985).

However, despite the availability of exemplary resources, access to quality professional development initiatives, and the endorsement of a broader range of pedagogical practices in State-sponsored curriculum documents (e.g., Curriculum Development Guidelines, Ministry of Education 1982-1985), there was little evidence of widespread change in mathematics classrooms (e.g., Blane & Clarke, 1983; Clarke, 1984; Clements, 1987; Lovitt, 1986). Although there were pockets of excellence, the majority of secondary mathematics textbooks looked much like those of the past and most mathematics lessons were focussed on demonstrating and practising routine procedures (Clarke, 1985). Committed as I was to approaching the teaching and learning of mathematics differently, I responded to an invitation to prepare a presentation on changing school mathematics at the *Essentials*

Revisited Workshop organised by the Mathematical Association of Victoria in October 1986. This resulted in an article (Siemon, 1987) in which I drew on Fullan's (1986) guides for thinking about change constructively, in particular, his observation that change = learning.

> Finally, it should be stressed, successful change, i.e. [sic] successful implementation, is none other than *learning*, but it is the adults in the system who are learning along with or more so than the students. Thus, anything we know about how adults learn is useful in deciding what to do and what not to do in approaching change. (p. 32)

My concern was not with the models of what an expanded teaching repertoire might look like, nor with the eminently worthy goal of codifying and sharing the wisdom of practice (Lovitt & Clarke, 1988a; Shulman, 1986), but with the premise that underpinned these efforts, that is, that teachers would adopt the new approaches if they tried them and saw that they 'worked'. In this context, teachers were seen as "constructivist learners, reflecting on their own practice, taking risks and attaining personal insights and personal growth as a result" (Lovitt & Clarke, 1988a, p. 3). However, the point I wanted to make in the article was that if the 'try it and see' experience was too far removed from teachers' knowledge and beliefs about what they felt they had to teach and how they perceived their role as teachers of mathematics, it was unlikely to be reflected upon in a way that would lead them to accommodate the particular practice into their repertoire. Teachers in this context do need to be seen as learners but as learners who come to any learning experience with knowledge and beliefs about the nature of mathematics, the teaching and learning of mathematics, and their roles as teachers of mathematics which mediate what is 'seen' and understood as a result of a 'try it and see' experience. I concluded that, in addition to working on how mathematics was taught, it was also necessary to engage with what was taught and what was valued – two tasks that in many ways were beyond the remit of practicing teachers but served to shape my long-standing interest in curriculum development and assessment.

In 1988 I presented my first MERGA keynote address. It was titled *Knowing and believing is seeing: A constructivist's perspective of change*. I am indebted to Ken Clements and the organising team at Deakin University in Geelong for the huge leap of faith they demonstrated in inviting a brash, relatively young PhD student to undertake this task, but it proved to be an extremely valuable stimulus to my thinking at the time. The conference was held at Deakin in Geelong and to the best of my knowledge, no conference proceedings were produced that year. Rather, Ken Clements and Neridah Ellerton edited a book titled, *School mathematics: The challenge to change* (Clements & Ellerton, 1989) that included a number of chapters based on contributions to the 1988 conference. I had derived my title from an Ashleigh Brilliant cartoon with the caption "SEEING IS BELIEVING: But I wouldn't have seen it if I hadn't believed it" (1987, p.40) and used a series of Australian, starting with one of the earliest colonial paintings, Thomas Watling's *Direct North General View of Sydney Cove* painted in 1794, and finishing with *Morning Star Myth* by Daninyawui of the Djanbabingu Tribe in East Arnhem Land, to show how the artist's knowledge, beliefs, and experience had shaped what they 'saw' and were able to represent of the Australian landscape. This metaphor was used to make the point that what teachers of mathematics know and believe about mathematics, the teaching and learning of mathematics, and their task as teachers of mathematics, powerfully shape their perceptions and motivates their decisions. To demonstrate the impact of this on practice, evidence was presented to suggest that teachers held personal and public theories of mathematics education that were not necessarily consistent, and that robust personal theories often prevailed over public theories in the context of day-to-day classroom practice (Siemon, 1989).

It seems strange now, but the role that teachers' knowledge and beliefs might play in teachers' pedagogical decision making was just beginning to be recognised as an object of systematic research (e.g., Shavelson & Stern, 1981; Thompson, 1984). A key influence on my thinking and subsequent research was the work on Cognitively Guided Instruction [CGI] (e.g., Fennema, Carpenter & Peterson, 1986) which was focussed on analysing and sharing evidence of students' mathematical thinking in relation to addition and subtraction word problems. This work demonstrated that where teachers were supported to develop detailed knowledge about their students' mathematical thinking they were motivated to evaluate and reconceptualise their pedagogical decisions. The importance the researchers attributed to teacher knowledge and beliefs in teachers' decision making is shown in Figure 1.

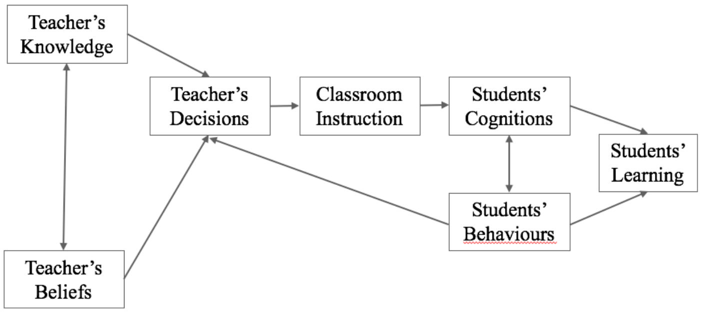

Figure 1. CGI Model for curriculum development (Fennema, Carpenter & Peterson, 1986)

In Australia, the efforts to codify and share the wisdom of practice were driven by practitioners for practitioners on the premise that a change in behaviour would lead to a change in beliefs (Lovitt & Clarke, 1988a), a process that Guskey (1985) described in his model of the processes involved in teacher change (see Figure 2). At the time, the focus on pedagogical elements such as concrete materials, story shells, technology, and investigations seemed a long way from international research on children's thinking and yet with the benefits of hindsight the two can be seen as essential complementarities. While the focus varies in terms of what was foregrounded and what was backgrounded both point to the ultimate role of teachers' knowledge and beliefs in the process of bringing about changes in classroom practice.

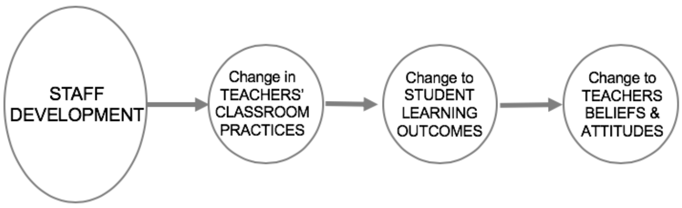

Figure 2. Guskey's (1985) model of the process of teacher change

What teachers know and why and how they know it are keys to change

Research on the nature of the knowledge needed to teach mathematics has grown significantly since the 1990s (e.g., Ball, 2000; Ball, Thames, & Phelps, 2008; Beswick, Callingham, & Watson, 2015; Copur-Gentuck, 2015: Chick, Baker, & Pham, 2006; Lee & Francis, 2018; Forgasz & Leder, 2008; Ma, 1999). Much of it emphasising the centrality of subject-specific content knowledge for teaching mathematics prompted by Shulman's (1986) observation that in educational research and policy circles there was a

> blind spot with respect to content … The emphasis is on how teachers manage their classrooms, organise activities, allocate time and turns, structure assignments, ascribe praise and blame, formulate the levels of their questions, plan lessons, and judge general student understanding. What we miss are questions about the *content* of the lessons taught, the questions asked, and the explanations offered. From the perspective of teacher development and teacher education, a host of questions arise. Where

> do teacher explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it, and how to deal with problems of misunderstanding? (emphasis in original, p. 8).

Building on Shulman's observation, Ball (2000) identified three problems that need to be solved to

> bring the study of content closer to practice and to prepare teachers to know and be able to use subject matter knowledge effectively in their work as teachers … The first problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how such knowledge needs to be held, and the third centres on what it takes to learn to use such knowledge in practice. (p. 244)

While these problems are not ones that are ever likely to be 'solved', it is my contention that the work on learning progressions/trajectories (LP/Ts) over the last two decades (e.g., Battista, 2004; Clarke & Clarke, 2004; Clements & Sarama, 2004; Maloney, Confrey, & Nguyen, 2014; Siemon, Breed, Izard, & Virgona, 2006; Siemon, Barkatsas, & Seah, 2019) has matured to the point where it can help us think about these problems. For example, the work on LP/Ts, has contributed to a much deeper understanding of the content knowledge that matters for teaching in terms of the likely paths that learners might take towards a particular learning goal, the important milestones and likely stumbling blocks they might meet along the way, and the indicative instructional activities to support them in that journey (Confrey, Maloney, & Nguyen, 2014; Siemon, 2019). In helping to identify the 'big ideas' that make a difference to student learning (e.g., Baroody, Cibulskis, Lai, & Li, 2004; Charles, 2005; Siemon, Bleckly, & Neal, 2012), LP/Ts bring greater coherence to a crowded curriculum and by identifying key benchmarks within progressions they offer a way in which teachers can hold this knowledge in their heads as they engage in complex classroom and school environments (e.g., Clarke & Clark, 2004; Confrey & Maloney, 2014; Siemon, Bleckly, & Neal, 2012). In relation to what it takes to learn to use such knowledge in practice, long-term LP/T studies have shown that where teachers have a detailed understanding of students' thinking they are more likely use this to adapt/modify their classroom practices in ways that are relevant to their particular circumstances (e.g., Bobis et al., 2005; Clarke, 2001; Fennema et al, 1996; Siemon, Banks, & Prasad, 2018; Timperley, Wilson, Barrar, & Fung, 2007; Timperley, 2011). This has important implications for professional development as noted in the *Scaling Up Innovative Practices in Mathematics and Sciences* Report (Carpenter et al., 2004).

> Learning with understanding depends on building on what students know and their ways of thinking. Similarly, the nature of instruction and professional development that we have studied is based on placing students' reasoning at the centre of instructional decision making. This not only represents a fundamental challenge to core educational practice, but it also represents a fundamental change in how we conceive of professional development and how it travels to new settings. …We have found that what travels – and can be sustainable — are patterns of reasoning and what teachers do with them, not the enactment of specific instructional activities (pp. 14-15).

My reason for raising *understanding change* as an issue at the intersection of research and practice is that while the extensive work on LP/Ts over the last two decades has made a significant contribution to what is known about the knowledge teachers need to teach mathematics, how they hold it, and what is takes to learn to use it in practice, there is more work to be done if we are to take seriously Fullan's (1986) adage that change = learning. And that is, understanding where teachers are in their professional learning journey and what they see as the 'job to be done' (Arnett, Moesta, & Horn, 2018; Clarke & Hollingsworth, 2002). Different teachers find themselves in different situations with different priorities.

> Sometimes they want their schools to improve. Sometimes they are looking for practical strategies and tools to make the classroom experience more engaging. Sometimes they struggle with feeling

powerless to meet the individual needs of every student. And sometimes they want to keep from falling behind on a school-wide initiative that has little appeal to them otherwise (Arnett, Moesta, & Horn, 2018, p. 6).

There is no shortage of good ideas and practical strategies for improving mathematics teaching and learning. Offered by practitioners and researchers alike and accessible through professional development and the publications and conferences of professional associations, they are also the subject of extensive research efforts. For example, the work on challenging tasks (e.g., Clarke & Roche, 2018; Sullivan et al, 2014; Zaslavsky, 2008), classroom organisations and cultures (e.g., Boaler, 2008; Rollard, 2012), teacher inquiry communities (e.g., Jaworski, 2006), teacher noticing (e.g., Mason, 2002; Lee & Francis, 2018), and teacher moves (e.g., Ellis, Özgür, & Reiten, 2019; Evans & Dawson, 2017). It is vital that teachers' have access to high quality, innovative, pedagogical practices, but the issue is and always has been one of scale. What 'works' in one context will not necessarily travel to another.

What is needed is a more coherent framework or way of thinking about professional development that respects and provides for teachers' perceptions of the 'job to be done' while at the same time offering the possibility of deepening their understanding of students' reasoning so that they are better equipped to "adjust instruction to their students' needs and understandings" (Carpenter et al., 2004, p. 10). In 2002, Clarke and Hollingsworth proposed an interconnected, non-linear model of professional development to help explain the "idiosyncratic and individual nature of teacher professional growth" (p. 965) within the change environment (see Figure 3).

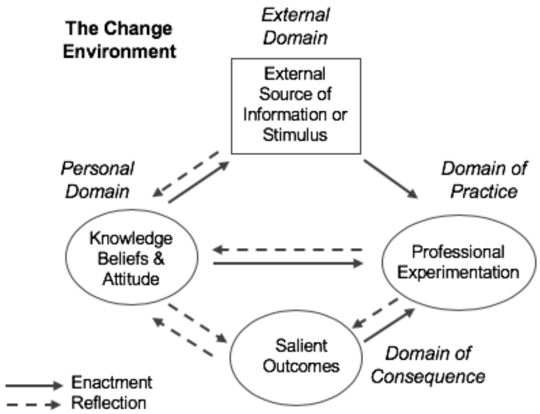

Figure 3. The Interconnected Model of Professional Growth (Clarke & Hollingsworth, 2002)

Based on Guskey's (1985) linear model, the interconnected model recognises different starting points and routes to professional growth and posited enactment and reflection as the "mechanisms by which change in one domain leads to change in another" (p. 950). However, in its present form the interconnected model does not appear to accommodate the type of professional growth generated by a detailed knowledge of students' mathematical thinking (e.g., Fennema, Carpenter & Peterson, 1986; Carpenter et al., 2004). Given the critical role of teachers in implementing and sustaining any change initiative and in light of the subsequent work on LP/Ts, it may well be worth revisiting this model and the one offered by Timperley (2009) shown in Figure 4 to examine their potential to support a deeper, more coherent, and nuanced understanding of what is needed to support teachers implement and sustain an unrelenting focus on student reasoning.

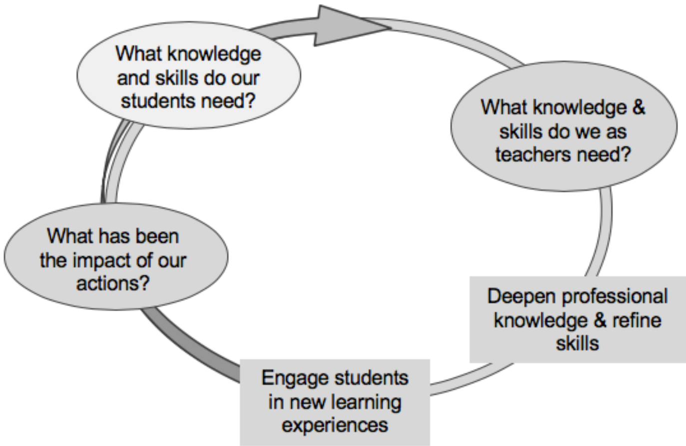

Figure 4. Teacher inquiry and knowledge-building cycle to promote valued student outcomes (Timperley, 2009)

The use (abuse?) of 'data'

At the outset I referred to the deification of 'research' as an issue in applying what is known to practice. The same claim could be made about 'data'. While there is an extensive body of research that points to the efficacy of data-informed educational decision making (e.g., Black & Wiliam, 1998; Hattie, 2012; Sharratt & Fullan, 2012), much of which is highly qualified and nuanced, 'data' have become deified to the point where teachers, schools, and systems feel compelled to collect and display these on data walls, some of which occupy whole rooms from floor to ceiling. Of course, as Hattie (2012) has pointed out it is not the data, nor the reports of the data, but the "professional judgements and consequences of the key person in the student learning debate over whom we have some influence, the teacher" (p. 173). And herein is the issue, not all of the data collected are fit for purpose, identifying what is relevant, interpreting what it means, and making judgements about where to go next and how to get there, are far from straightforward. These are significant tasks requiring a deep understanding of curriculum content for teaching (pedagogical content knowledge), an appreciation of the underlying ideas that are likely to make a difference, and a familiarity with and confidence to use the sort of instructional tasks and strategies that will promote student learning.

A related issue is the form of data collected. Teachers, school leaders, and educational systems are awash with data courtesy of the digital revolution which makes it possible to collect and assess 'data' in real time. But this severely restricts the type of data collected and rarely, if ever, reveals student thinking without detailed item analyses. Data carry a sense of authenticity, objectivity, and an imperative to respond, and yet much of the data that school leaders and teachers are exhorted to use are simply too broadly specified or tied to relatively unimportant aspects of curriculum to support the sort of professional discussions referred to by Hattie.

> Common assessments are the foundation of a data wall – providing the achievement yardstick to measure student growth. These can be diagnostic (like unit pre-tests), formative (running records) or summative (NAPLAN). Whatever the type, common assessments should address a Victorian Curriculum achievement standard linked to an agreed learning goal along the continuum of student learning (whole-school, year- or learning area-specific) (Victorian Department of Education, p. 2).

The Role of Formative Assessment

Teaching informed by quality assessment data has long been recognised as an effective means of improving mathematics learning outcomes (e.g., Black & Wiliam, 1998; Goss, Hunter, Romanes & Parsonage, 2015; Masters, 2013; National Council of Teachers of Mathematics, 2001; Timperley, 2009; Wiliam, 2011). It is also evident that where teachers are supported to identify and interpret student learning needs, they are more informed about where to start teaching, and better able to scaffold their students' mathematical learning (Callingham, 2010; Clarke, 2001; Siemon, 2016). Furthermore, Wiliam (2006) wrote:

> What we *do* know is that when you invest in teachers using formative assessment … you get between two and three times the effect of class size reduction at about one-tenth the cost. So, if you're serious about raising student achievement … you have to invest in teachers and classrooms, and the way to do that is in teacher professional development focused on assessment for learning. (p. 6)

It is beyond the scope of this paper to embark on an extended discussion of the nature of formative assessment suffice to say that "any assessment is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction" (Wiliam, 2011, p. 43). With that in mind and with a view to defining formative assessment in a way that was useful to practitioners, Wiliam (2013) suggested that it would be useful to think in terms of:

1. Where the learner is right now
2. Where the learner needs to be
3. How to get there.

Of course, this begs the question of *where* in relation to *what*. If the 'where' is defined in terms of Curriculum outcome statements, system-level assessments such as the National Assessment Program for Literacy and Numeracy (NAPLAN) are unlikely to be fine-grained enough to provide the sort of data that teachers could use to identify the specific learning needs of any one student. On the other hand, these data can be useful at the school or system level if they prompt consideration of a broader range of strategies aimed at improving student achievement in numeracy such as additional resources, and/or investment in targeted professional development. And as Callingham (2010) has pointed out, this type of data can also be used formatively where, for instance, a particular item and the possible responses are unpacked and discussed at the classroom level. Classroom assessments generated by a teaching team are likely to be more fine-grained than system-level assessments because they generally assess a much smaller range of outcomes (e.g., a pre-test on subtraction at Year 3). However, depending on the form of such assessments, the data they produce may not help teachers decide where to start teaching unless they have a detailed understanding of students' reasoning in the domain including the likely sources of difficulty and an appreciation of the types of activities needed to progress students' learning. Using either of these forms of data as a basis for grouping students into ability groups (e.g., low, middle, and high) is in my view completely indefensible. Quite apart from the known and well-researched issues with ability grouping (e.g., Boaler, 2005; Linchevksi & Kutscher, 1998; Zevenbergen, 2003), students displaying 'similar' levels of performance may do so for completely different reasons.

Alternatively, if the *where* is based on what research suggests is most likely to make a difference, that is, big ideas such as trusting the count, place-value, multiplicative thinking and equipartitioning (Confrey, Maloney, Nguyen, & Rupp, 2014; Siemon, Bleckly, & Neal, 2012), and the assessments reveal student thinking, there is a substantial body of evidence to suggest that targeted teaching will result in improved student outcomes (Goss, Hunter, Romanes, & Parsonage, 2015; Siemon, 2016, 2017b). To illustrate this, I would like to draw on a case study that was reported in Siemon, Banks and Prassad (2018). It concerns one of the schools that participated in the *Reframing Mathematical Futures II* (RMF II) project from late 2014 to 2018 (see Siemon, 2017a). Plumpton High School (name used with permission) is a large multi-cultural 7 to 12 secondary school in an established outer suburb of one of Australia's largest cities. The school prioritises English, Maths and Science and one of its key goals is to "put students first". In late 2014, staff were concerned about the mathematics results and the declining number of students electing to pursue the more advanced maths courses in the senior years. As one of the 'new' schools that had not participated in the earlier Priority project (see Siemon, 2016), the school was supported to implement a targeted teaching approach to multiplicative thinking in 2015 prior to participating in the mathematical reasoning component of the RMF II project from 2016. The 'new' schools were introduced to the formative assessment resources for multiplicative thinking produced by the *Scaffolding Numeracy in the Middle Years* (SNMY) project (Siemon, Breed, Dole, Izard, & Virgona, 2006) at an introductory residential workshop where they also had the opportunity to learn from the experience of the Priority project schools. The SNMY materials include an evidenced-based Learning and Assessment Framework (LAF) for multiplicative thinking comprising eight developmental Zones and two validated assessment options that map each student's performance to one of the eight Zones. To give teaching staff time to prepare the targeted teaching activities over the summer break, SNMY Assessment Option 1 was administered to all Year 7 students in December 2014. The school leadership supported the decision to focus on Year 8 in 2015 as this cohort would sit the NAPLAN test in Year 9 in 2016 which would provide an independent evaluation of the intervention.

In 2015, each of the six Year 8 classes had a dedicated 75 minutes 'RMF' lesson per week. During this time, the students worked in their Zone groupings on targeted teaching activities linked to the LAF. The school-based RMF II contact person and another senior maths teacher, visited the classrooms whenever they could and prepared resources in their free periods. As time went on and the demand for new, age-appropriate activities increased, the Year 8 teachers also developed and shared Zone-based activities with their colleagues. One of the ways in which this happened was at the Wednesday lunches, where Year 8 staff talked about what they were doing, reflected on progress and developed new ideas. A lesson template was developed, and staff would workshop new lessons prior to delivery. Referred to as 'Live in Lessons', this enabled the team to iron out any potential issues and to make links to regular classroom teaching activities and content. Priority was given to purchasing concrete materials and a separate area was set up to keep class booklets, resources and activities for easy collection and distribution.

From everyone's perspective it was a tough year. There was little buy-in from students and teachers at the outset as working in groups was something new for many. The existing class structure (semi-streamed) helped manage the targeted teaching approach to some extent but there was still considerable variation within each classroom. Planning was essential and, in retrospect, it was a key factor to the school's success. Over the course of the year, teachers found that they were incorporating many of Zone-type activities into the curriculum being taught in the week, placing particular emphasis on the need to explain and justify solution strategies as this had proved to be a major sticking point early on. The team learnt as they went and kept on sharing, adjusting and implementing strategies/activities which worked in other classes. Staff meetings on Mondays were focussed on developing teachers' capacity to share resources and ideas to help the growth of targeted teaching in classrooms.

Gradually, everything became easier, the students were more accustomed to working in groups and appreciated the opportunity to experience success. Student engagement increased and the quality of their responses to school-based assessments improved noticeably. Teaching staff were more inclined to design reasoning activities for regular classroom teaching and provide time for students to apply what they know in unfamiliar contexts and marking rubrics were slowly incorporated into classroom assessment tasks.

SNMY Assessment Option 2 was administered to all Year 8 students in September 2015 and subsequently marked and moderated by the Year 8 teachers and the two senior maths teachers. The results were impressive and immediately bought buy-in from senior management and other maths teachers. Additional teacher release was provided to support the preparation of resources, marking and moderating of assessments, and training of other staff members. In December 2014, 52% of the Year 7 cohort were in Zones 1 to 3 of the LAF. By September 2015, only 30% were in Zones 1 to 3. In 2014, only 16% of the students were in Zones 6 to 8. In 2015, this had risen to 40%. The growth is shown in Figure 5 and represents an effect size of over 1.

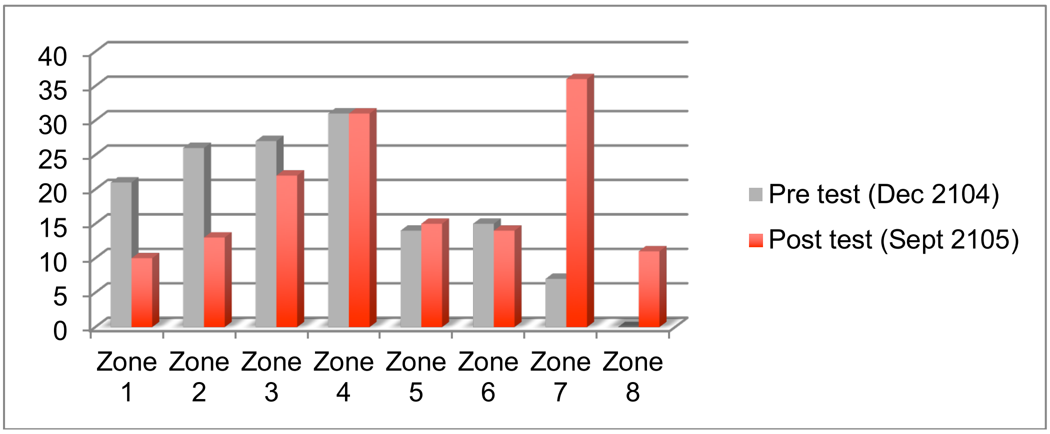

Figure 5. Percentage of students in each Zone of the SNMY LAF in August 2014 (n = 141) and September 2015 (n = 152) (November 2016)

While not the only measure of success in school mathematics, the Year 9 NAPLAN results for the same cohort in 2016 provide conclusive evidence that targeted teaching makes a difference. Compared to the previous Year 9 who sat the NAPLAN test in 2015, the average scaled growth score for the school went from below all State in 2015 (45.6) to above all State in 2016 (51.1) (Siemon, Banks, & Prasad, 2018).

Knowledge is power

As indicated above, identifying what students know in relation to what makes a difference to students' learning and acting on that is not straightforward, it requires assessment tasks that elicit student thinking and, as we learnt from the *Supporting Indigenous Student Achievement in Numeracy* project (Commonwealth of Australia, 2005), it requires interpretations of what different student responses might mean and targeted teaching advice linked to those specific responses to support teachers build on what students know. While the project was initially framed in terms of exploring the use of negotiated rich tasks with a view to improving numeracy outcomes, it quickly became apparent that the use of such tasks in remote communities was inappropriate. On speaking with the school leaders, teachers, and the Indigenous elders involved in the schools, it was clear that what they wanted was for their children to experience success in 'two worlds' (i.e., they wanted their children to learn school mathematics to as well as become proficient in their own language and culture).

During one of my visits to Millingimbi I had the opportunity to discuss this situation with Ganygulpa, a respected Indigenous elder, during which I suggested that we trial a number of the performance-based interview tasks that I had developed as part of the assessment requirements for pre-service primary students undertaking the one-year Graduate

Diploma of Education (Primary) program at RMIT University, to see if we could identify starting points for teaching 'school mathematics'. The tasks were based on key aspects of the primary mathematics curriculum (e.g., place value) and were drawn from tasks reported by researchers working in the field (e.g., Ross, 1989). The tasks did not require students to read or write in English but their 'performance' (physical actions and/or oral description in first language or English) could be interpreted in terms of different levels of understanding. When I suggested that perhaps we could think about the Indigenous teacher assistants presenting the tasks in first language the results of which they could share with the trained, non-Indigenous classroom teacher, she smiled, thought for a while and said … "Mäny mak (good) that means we could choose who to tell". The reason I tell this story in this context is that data in the form of evidence about children's mathematical thinking can be a powerful force in the right hands – in this case the data had the potential to redress the power imbalance in classrooms where the trained, classroom teachers were often dismissive of the Indigenous adult in the room assigning them menial jobs such as translating instructions or restoring order. The probe tasks as they came to be called together with their associated advice were subsequently trialled and found to be effective in identifying starting points for teaching school mathematics in remote Indigenous communities. The process also contributed to the developing pedagogical content knowledge of the Indigenous adults involved (Commonwealth of Australia, 2005).

Where quality information about students' reasoning is elicited, interpreted and acted upon it can lead to improvements in student learning but where the information collected is limited to the number of exercises completed in a digitally-based mathematics program, a ranking based on a comprehensive test that may or may not have assessed what was taught, or performance on any form of assessment that does not elicit students' mathematical reasoning, its use for formative assessment of the type envisaged by Wiliam (2013) is highly questionable. Research has a powerful role to play in identifying what information counts, the forms of assessment most likely to elicit students' mathematical reasoning, and together with practitioners, the sort of instructional strategies likely to progress students' learning. But it is the first of these that is most difficult to negotiate and apply in practice.

Curriculum Development

Who gets to decide what should be taught when is a vexed issue at the intersection of research and practice. While the work on LP/Ts has a lot to contribute to this debate, the processes surrounding curriculum development in Australia have inevitably led to compromise with little indication of those areas that are more important for future learning than others and descriptors framed in terms of what can be easily measured rather than student reasoning. Given that implementing 'the curriculum' is seen by teachers as an important aspect of the 'job to be done', how it is framed and supported inevitably shapes the design of instructional strategies and assessments. In recent years, and largely in response to the narrowing focus of the curriculum, there have been calls for an increased focus on 'big ideas' in mathematics teaching and learning (e.g., Baroody, Cibulskis, Lai, & Li, 2004; Charles, 2005; Ma, 1999; Siemon, 2006) and for much greater coherence and alignment between curriculum, instruction, and assessment (Black, Wilson & Yao, 2011; Pellegrino, 2008; Swan & Burkhardt, 2012; Wilson, 2018). The value of adopting a 'big ideas' approach to planning are exemplified in the experience of River Gum Primary School (name used with permission). It is also a nice illustration of what can be accomplished by researchers, school leaders, and teachers working together at the interface of research and practice.

A process of co-construction

As indicated above, one obvious arena where mathematics education research can contribute to practice is the design and implementation of school mathematics curricula. This observation holds whether we are talking about curriculum as a set of broad, measurable competencies (i.e., content descriptors) or as a comprehensive set of resources for teaching and learning mathematics. In Australia, provided schools base their planning decisions on the *Australian Curriculum: Mathematics* (Australian Curriculum Assessment & Reporting Authority, 2019), schools have considerable leeway in deciding exactly what they will teach, when, and how they will teach it - however, responsibility for this tends to rest with teaching teams rather than individual teachers.

River Gum Primary School is located in an outer metropolitan suburb of a major Australian city. The school has approximately 50 staff and 500 students. The school is located in a low socio-economic area with links to 58 different cultural groups. Over 75% of the families come from non-English speaking backgrounds. In 2009 a new Principal was appointed and the revitalisation of the school leadership team led to a sustained period of school improvement that started with a strong shared vision and a focus on building staff capacity to deliver that vision. A critical first step was the preparation of pedagogical master plan for the school, a process that helped bring about a school-wide commitment to personalised learning within a community of learners. A decision was made to begin with a focus on improving literacy teaching and learning using Boushey and Moser's (2006) *Daily 5 Model*, which aims to build independent learning behaviours and engagement through the negotiation of agreed social norms and the provision of choice in how each of the five 'must do's are undertaken each day.

In 2011, half-way through the first year of their three-year school improvement plan, the leadership team were looking for ways to support the second year of the plan which was focused on improving mathematics outcomes. With the support of the newly appointed part-time Numeracy Teaching and Learning Coach, the leadership group and subsequently the whole staff participated in a series of professional learning sessions on the use of the *Assessment for Common Misunderstanding* (AfCM) formative assessment materials (Siemon, 2006). The materials were derived from the probe tasks and teaching advice used in the SISAN project but their grouping into non-negotiable 'big ideas' by key stages in schooling was based on the evidence obtained from the SNMY project. For this purpose, a 'big idea' was defined as a key aspect or way of thinking about mathematics without which, students' progress in mathematics will be seriously impacted; that connects other ideas and strategies; provides an organising structure to support further learning and generalisations; and can be observed in activity (Siemon, 2006).

Following the professional learning sessions, teachers were invited to interview a number of children using one or two of the tools (i.e., performance-based tasks) and to share their observations in teaching teams. Initially the teachers were shocked at the results, but this firmed their resolve to find a way to incorporate the use of the materials into their practice in a systematic way that reflected the successful features of the Daily 5 model. Having recently completed the *Developmental Maps for Number* (Siemon, 2010), which were based on the AfCM advice but offered a more detailed indication of the key ideas and strategies underpinning the development of the big ideas, I shared the maps with the leadership team and asked if these would suit the school's purpose. The leadership group felt they were "perfect for their needs" and could see how they might be adapted to address the gaps in student learning and teacher capacity. A month later, a partnership was formed whereby I would meet with the leadership team on a semi-regular basis and the school would trial the use of the maps to introduce a more targeted and individual approach to the teaching and learning of mathematics. In 2012, two of the leading teachers were provided with significant time release, which they used to assess all Year 3 to 6 students on the Trusting the Count and Place Value tools and develop a student-friendly version of the maps. The results were discussed first with teaching teams and then by the whole staff which led to the decision to use the maps to support a targeted teaching approach throughout the school. The student friendly maps were subsequently introduced in the Preparatory to Year 2 classrooms to support a student-centred approach to learning mathematics based on the Daily 5 model. Since that time and apart from one or two professional development sessions to introduce new teachers to the formative assessment materials, the school has based its planning on the understandings they have gained about students' thinking in relation to the big ideas. While the results they have achieved are impressive showing consistent improvement on NAPLAN result compared to like schools, the deep knowledge that the teachers have gained as a result of "grounding their professional learning in the immediate problems of practice" (Timperley, 2011, p. 3) has enabled them to persist and adapt the targeted teaching approach to suit their needs. The curriculum still serves as a guide to their everyday planning, but it is seen through the lens of the big ideas, which serves as a filter to ensure instructional priority is given to those aspects of the mathematics curriculum that are most likely to make a difference to student learning and confidence.

Conclusion

There is much we can learn from well-documented images of best-practice, research that investigates what works and why, and from research-practice partnerships that explore students' mathematical thinking and use this as a basis for planning. But to support sustained changes in how mathematics is taught and learnt, these efforts need to be more closely aligned and the impediments to change – skills-focussed, high-stakes assessments and cryptic, non-aspirational curriculum documents – need to be challenged alongside or ahead of efforts to create rich learning environments that draw on the wisdom of practice but "place students' reasoning at the centre of instructional decision making" (Carpenter et al., 2004, p. 10). This is no easy task as scaling up studies have shown, it requires ongoing professional learning opportunities for teachers to deepen their knowledge of students' reasoning in ways that enable them to recognise and build on what students' know and adapt their instructional decisions to meet the changing needs and circumstances of their particular students. Beyond the level of the classroom it requires researchers working with educational leaders and systems to bring about a greater alignment between curriculum, instruction, and assessment. The exciting thing is that significant progress has and is being made. By identifying probable pathways in students' learning and by bringing together and drawing on what we know about effective instructional strategies and quality forms of assessment, evidence-based LP/Ts offer "an unprecedented degree of coherence among standards, assessment, … instruction and curriculum" (Confrey & Maloney, 2014, p. 134) and go a long way towards addressing the issues raised here at the intersection of research and practice.

References

Arnett, T., Moesta, B., & Horn, M. (2018). *The teachers's quest for progress: How school leaders can motivate instructional innovation.* Christenson Institute. https://eric.ed.gov/?id=ED593236

Australian Curriculum Assessment & Reporting Authority. (2019). *Australian Curriculum: Mathematics*. https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/

Ball, D. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51*(3), 241-247.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Baroody, A., Cibulskis, M., Lai, M., & Li, X. (2004). Comments on the issue of learning trajectories in curriculum development and research. *Mathematics Teaching and Learning, 6*(2), 227-260.

Barton, W. (2009). Being mathematics, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides*. (Proceedings of the 32nd annual conference of the Mathematical Research Group of Australasia, Vol. 1, pp. 3-9). Auckland, NZ: MERGA.

Battista, M. (2004). Applying cognition-based assessment to elementary school students' development of understanding of area and volume measurement. *Mathematical Thinking and Learning, 6*(2), 184-204.

Beesey, C. (1989). Exploring mathematics in classrooms. In B. Doig (Ed.) *Everyone counts*. (Proceedings of the 26th annual conference of the Mathematical Association of Victoria, pp. 108-110). Melbourne: MAV.

Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 15*(2), 131-157.

Blane, D., & Clarke, D.J. (1983). The realities and essentials of teacher training. In D. Blane (Ed.) *The essentials of mathematics education* (Proceedings of the 20th annual conference of the Mathematical Association of Victoria). Melbourne: MAV.

Black, P., & Wiliam, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappa*, *80*(2), 139–147.

Black, P., Wilson, M., & Yao, S. (2011). Road maps for learning: A guide to the navigation of learning progressions. *Measurement: Interdisciplinary Research and Perspectives*, *9*, 71–123.

Boaler, J. (2005). The 'psychological prisons' from which they never escaped: The role of ability grouping in reproducing social class inequalities. *Forum, 47*(2-3), 135-143.

Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*(2), 167–194.

Bobis, J., Clarke, B.A., Clarke, D. M., Thomas, G., Wright, R., Young-Loveridge, J., & Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27-57.

Boushey, G., & Moser, J. (2006). *The daily five. Fostering literacy independence in the elementary grades*. Portland, Maine: Stenhouse Publishers.

Brilliant, A. (1987). *I may not be perfect – But parts of me are perfect.* California: Woolbridge Press.

Callingham, R. (2010). Mathematics assessment in primary classrooms. In ACER (Ed.), *Proceedings of 2010 ACER Research Conference* (39-42), Melbourne: ACER.

Callingham, R. (2011). Mathematics assessment: Everything old is new again? In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics traditions and (new) practices* (Proceedings of the 34th annual conference of the Mathematical Education Research Group of Australasia and the Australian Association of Mathematics Teachers, Vol. 1, pp. 3-10). Alice Springs: MERGA.

Carpenter, T., Blanton, M., Cobb, P., Loefe Franke, M., Kaput, J., & McClain, K. (2004). *Scaling up innovative practices in mathematics and science*. Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science.

Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Education Leadership, 7(*3), 9-24.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers; pedagogical content knowledge for decimals. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education* (Vol.2, pp.297-304). Prague: PME.

Clarke, B.A., & Clarke, D.M. (2004). *A framework of growth points as a powerful professional too*l. Paper presented to the annual meeting of the American Educational Research Association, San Diego, April.

Clarke, D. J. (1984). Secondary mathematics teaching: Towards a critical appraisal of current practice. *Vinculum, 21*(4), 16-22.

Clarke, D. J. (1985). The impact of secondary schooling and secondary mathematics on student mathematical behaviour. *Educational Studies in Mathematics, 16*(3), 231-257.

Clarke, D. J. (2005). Essential complementarities: Arguing for an integrated approach to research in mathematics classrooms. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice*. (Proceedings of the 28th annual conference of the Mathematical Research Group of Australasia, Vol. 1, pp. 3-17). Geelong Vic: MERGA.

Clarke, D. J., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*(8), 947-967.

Clarke, D.M. (2001). Understanding, assessing and developing young children's mathematical thinking: Research as a powerful tool for professional growth. In J. Bobis, B. Perry & M Mitchelmore (Eds.), *Numeracy and beyond* (Proceedings of the 24th annual conference of the Mathematics Education Group of Australasia, pp.9-26). Sydney: MERGA.

Clarke, D.M., & Roche, A. (2018). Using contextualized tasks to engage students in meaningful and worthwhile mathematics learning. *Journal of Mathematical Behaviour, 51*, 95-108.
Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning, 6*(2), 81-89.
Clements, M. (1987). Personally speaking. *Vinculum, 24*(4), 3-4.
Clements, M. (2012). MERGA 2012: Where we've been, where we are, and where we're going. In J. Dindyal, L Cheng, & S. Ng (Eds.), *Mathematics education: Expanding horizons*. (Proceedings of the 35th annual conference of the Mathematical Education Research Group of Australasia, Vol. 1, pp. 4-19). Singapore: MERGA.
Clements, M., & Ellerton, N. (1989). *School mathematics: The challenge to change*. Geelong, Australia: Deakin University Press.
Commonwealth of Australia. (2005). *Supporting Indigenous students' achievement in numeracy*. Canberra, ACT: Commonwealth of Australia.
Confrey, J. & Maloney, A. P. (2014). Linking standards and learning trajectories: Boundary objects and representations. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 125-160). Charlotte: Information Age Publishing.
Confrey, J., Maloney, A., Nguyen, K., & Rupp, A. (2014). Introduction. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. i-xiv). Charlotte: Information Age Publishing.
Confrey, J., Maloney, A., Nguyen, K., & Rupp, A. (2014). Equipartitioning, A foundation for rational number reasoning: Elucidation of a learning trajectory. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 61-96). Charlotte: Information Age Publishing.
Copur-Gencturk, Y. (2015). The effects of changes in mathematical knowledge on teaching: A longitudinal study of teachers' knowledge and instruction. *Journal for Research in Mathematics Education, 46*(3), 280-330.
de Mestre, N. & Duncan, B. (1980). *Sixty tasks from the ACT Mathematics Centre*. Canberra: Curriculum Development Centre.
Ellis, A., Özgür, Z., & Reiten, L. (2019). Teacher moves for supporting student reasoning. *Mathematics Education Research Journal, 31*(2), 107-132.
English, L. (1984). What do you do when you don't know what to do? Problem solving in early childhood. In A. Maurer (Ed.), *Conflicts in mathematics education* (Proceedings of the 21st annual conference of the Mathematical Association of Victoria). Melbourne: MAV.
Evans, S. & Dawson, C. (2017). Orchestrating productive whole class discussions: The role of designed student responses. *Mathematics Teacher Education and Development, 19*(2), 159-179.
Forgasz, H. (2008). Stars, compass, and GPS: Navigating currents and charting directions for mathematics education research on gender issues. In M. Goos, R. Brown, & K. Mokar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematical Education Research Group of Australasia, Vol. 1, pp. 5-15). Brisbane: MERGA.
Forgasz, H., & Leder, G. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan & T. Wood (Eds.), *Handbook of mathematics teacher education: Vol 1. Knowledge of and beliefs in mathematics teaching and teaching devel*opment (pp. 173-192). Rotterdam, The Netherlands: Sense Publishers.
Fennema, E., Carpenter, T.P, Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*, 403-434.
Fennema, E., Carpenter, T., & Peterson, P. (1986). *Teachers' decision making and cognitively guided instruction*. Paper presented to the 10th annual conference of the International Group for the Psychology of Mathematics Education. London, July.
Fullan, M. (1986). Improving the implementation of educational change. *School Organisation, 6*(3), 321-326.
Gailbraith, P. (2014). Custodians of quality: Mathematics education in Australasia. Where from? Where at? Where to? In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematical Education Research Group of Australasia, Vol. 1, pp. 38-53). Sydney: MERGA.
Geher, G. (2016). Black-and-white thinking in our social worlds – The evolutionary basis of simple thinking. *Psychology Today* (January). Retrieved from https://www.psychologytoday.com/au/blog/darwins-subterranean-world/201601/black-and-white-thinking-in-our-social-worlds
Goss, P., Hunter, J., Romanes, D., & Parsonage, H. (2015). *Targeted teaching: How better use of data can improve student learning*. Melbourne: Grattan Institute.
Guskey, T. R. (1985). Staff development and teacher change. *Educational Leadership, 42*(7), 57–60.

Hattie, J. (2012). *Visible learning for teachers: Maximising impact on learning*. Abingdon, UK: Routledge.
Horne, M. (1984). Some problem solving. In A. Maurer (Ed.), *Conflicts in mathematics education* (Proceedings of the 21st annual conference of the Mathematical Association of Victoria). Melbourne: MAV.
Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*, 187-211.
Lee, M, & Francis, D. (2018). Investigating the relationships among elementary teachers' perceptions of the use of student thinking, their professional noticing skills and their teaching practices. *Journal of Mathematical Behaviour, 51*, 118-128.
Linchevski, L. & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed ability versus same-ability grouping in mathematics, *Journal for Research in Mathematics Education, 29*(5) 533-554.
Lovitt, C. (1986). Giving meaning to the jargon in the classroom. In A. Maurer (Ed.) *Conflicts in mathematics education,* (Proceedings of the 21st annual conference of the Mathematical Association of Victoria, pp. 415-419), Melbourne: MAV.
Lovitt, C., & Clarke, D. (1986). *MCTP first year report.* Canberra: Curriculum Development Centre.
Lovitt, C., & Clarke, D. (1988a). Codifying and sharing the wisdom of practice. In J. Pegg (Ed.), *Mathematical interfaces* (Proceedings of the 12th biennial conference of the Australian Association of Mathematics Teachers). Newcastle, NSW: AAMT.
Lovitt, C. & Clarke, D, M. (1988b). *The Mathematics Curriculum Teaching Program – Professional development package. Activity Banks I and II*. Melbourne: Curriculum Development Centre.
Lowe, I. (1980). Teaching for problem solving with RIME. In P. Williamson (Ed.) *Learn to love mathematics* (Proceedings of the 17th annual conference of the Mathematical Association of Victoria). Melbourne: MAV.
Lowe, I., & Lovitt, C. (1984). R.I.M.E. In A. Maurer (Ed.) *Conflicts in mathematics education,* (Proceedings of the 21st annual conference of the Mathematical Association of Victoria, pp. 406-414), Melbourne: MAV.
Lowrie, T. (2015). Mathematics education as a field of research: Have we become too comfortable. In M. Marshamn, V. Geiger, & A. Bennison (Eds.), *Mathematics education at the margins* (Proceedings of the 35th annual conference of the Mathematical Education Research Group of Australasia, Vol. 1, pp. 14-24). Sunshine Coast: MERGA.
Ma, L. (1999). *Knowing and teaching elementary mathematics*. New York, NY: Routledge.
Maloney, A., Confrey, J., & Nguyen, K. (Eds.). (2014). *Learning over time: Learning trajectories in mathematics education* (pp. 125-160). Charlotte: Information Age Publishing.
Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: Routledge-Falmer.
Masters, G. (2013). Reforming educational assessment: Imperatives, principles and challenges. *Australian Education Review, No. 57*, 1-57.
National Council of Teachers of Mathematics [NCTM] (1980). *An agenda for action*. Washington, DC: NCTM.
National Council of Teachers of Mathematics [NCTM] (2001). *Principles and standards for school mathematics.* Reston, VA: NCTM.
Pellegrino, J. (2008, December). *The design of an assessment system for the race to the top: A learning sciences perspective on issues of growth and measurement*. Paper presented at the Exploratory Seminar: Measurement challenges within the race to the top agenda. Retrieved from https://pdfs.semanticscholar.org/53b7/6668fec653df7db1261304bd43a4ce64e42d.pdf
Richards, M. (1985). Problem solving task centres: What, why, how? *Vinculum, 22*(2), 3-5
Robinson, I. (1986). *Exploring mathematics in classrooms*. In N. Ellerton (Ed.) *Mathematics: Who needs, what?* (Proceedings of the 23rd annual conference of the Mathematics Association of Victoria). Melbourne: MAV.
Rollard, R. (2012). Synthesising the evidence on classroom goal structures in middle and secondary schools: A meta-analysis and narrative review. *Review of Educational Research, 82*(4), 396-435.
Ross, S. (1989). Parts, wholes, and place value: A developmental view. *Arithmetic Teacher, 36*, 47- 51.
Shavelson, R. & Stern, P. (1981). Research on teachers' pedagogical thoughts, judgements, decisions, and behaviour. *Review of Educational Research, 51*(4), 455-498.
Sharratt, L. & Fullan, M. (2012). *Putting FACES to data: What great leaders do*. Thousand Oaks, CA: Corwin Press.
Shulman, L. (1986). Those who understand: Knowledge growth in teaching, *Educational Researcher, 15*(2), 4-14.
Siemon, D. (1985). Problem Solving – Should we add on or integrate? In P. Sullivan (Ed.), *Mathematics curriculum, teaching and learning* (Proceedings of the 22nd annual conference of the Mathematical Association of Victoria). Melbourne: MAV

Siemon, D. (1987). Effective strategies for changing mathematics education, *Vinculum, 24*(4), 13-28.
Siemon, D. (1989). Knowing and believing is seeing: A constructivist's perspective of change. In M. Clements & N. Ellerton (Eds.). *School mathematics: The challenge to change*. Geelong, Vic: Deakin University Press.
Siemon, D. (2006). *Assessment for common misunderstandings materials*. Prepared for and published electronically by the Victorian Department of Education and Early Childhood Development. http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx
Siemon, D. (2010). *Developmental maps for number*. Commissioned by the Victorian Department of Education and Early Childhood Development to support the development of e-book boxes. Unpublished manuscript: RMIT University.
Siemon, D. (2016). Addressing the STEM challenge through targeted teaching: What's the evidence? *Proceedings of the 2016 ACER Research Conference* (pp. 74 – 78). Melbourne: ACER.
Siemon, D. (2017a). Developing learning progressions to support mathematical reasoning in the middle years: Introducing the Reframing Mathematical Futures II project. In A. Downton., S. Livy., & J. Hall (Eds.), *40 years on: We are still learning!* (Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 651-654). Adelaide: MERGA.
Siemon, D. (2017b). Targeting the 'big ideas' in mathematics. *Teacher Magazine*, 27 February, Retrieved from https://www.teachermagazine.com.au/article/targeting-big-ideas-in-mathematics
Siemon, D. (2019). Knowing and building on what students know – The case of multiplicative thinking. In D. Siemon, T. Barkatsas, & R. Seah (Eds.). *Researching and using learning progressions (trajectories) in mathematics education*. The Netherlands: Brill/Sense Publishing.
Siemon, D. Banks, N., & Prasad, S. (2018). Multiplicative thinking a STEM foundation. In T. Barkatsas, N. Carr & G. Cooper (Ed.), *STEM education: An emerging field of inquiry*. Sense Publications.
Siemon, D., Barkatsas, T., & Seah, R. (Eds.) (2019). *Researching and using learning progressions (trajectories) in mathematics education.* The Netherlands: Brill/Sense Publishing.
Siemon, D., Bleckly, J. & Neal, D. (2012). Working with the big ideas in number and the Australian Curriculum Mathematics. In W. Atweh, M. Goos, R Jorgensen & D. Siemon (Eds.) *Engaging the Australian Curriculum Mathematics – Perspectives from the field* (pp. 19-46). Online Book, MERGA. https://www.merga.net.au/Public/Public/Publications/Engaging_the_Australian_curriculum_mathematics_book.aspx
Siemon, D., Breed, M., Dole, S., Izard, J., & Virgona J. (2006). *Scaffolding Numeracy in the Middle Years – Project findings, materials and resources, Final report*. Retrieved from http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx
Southwell, B. (1982). The problem as communication. In J. Veness (Ed.), *Mathematics: A universal language* (Proceedings of the 9th biennial conference of the Australian Association of Mathematics Teachers, pp. 148-152). Sydney: AAMT.
Stacey, K., & Groves, S. (1985). *Strategies for problem solving*. Burwood: VICTRACC.
Sullivan, P., Askew, M., Cheeseman, J., Clarke, D.M., Mornane, A., Roche, A., & Walker, N. (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education, 18*, 123-140.
Swan, M., & Burkhardt, H. (2012). A designer speaks – Designing assessment of performance in mathematics. *Educational Designer,* 2(5), 4-41. Retrieved 14 February 2018 from: http://www.educationaldesigner.org/ed/volume2/issue5/article19/
Thompson, A. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), 105-127.
Timperley, H. (2009) Using assessment data for improving teacher practice. Paper presented to 14th annual Research Conference Australian Council for Educational Research, *Assessment and Student Learning: Collecting, interpreting and using data to inform teaching*. Perth, August.
Timperley, H. (2011). *Using student assessment for professional learning: Focussing on student outcomes to identify teachers' needs*, Paper Number 21. Victorian Department of Education & Early Childhood Development. https://www.education.vic.gov.au/Documents/about/research/timperleyassessment.pdf
Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration*. Wellington, NZ: Ministry of Education.
Victorian Department of Education (2018). Professional practice: Using data walls to turn data into instruction. https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/practice/Professional_practice_note_5_data_walls.pdf
Wiliam, D. (2006). *Assessment for learning – what, why and how*. Edited transcript of a presentation to the Cambridge Assessment Network Conference (September). University of Cambridge, UK.

Wiliam, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree Press.
Wiliam, D. (2013). Assessment: The bridge between teaching and learning. *Voices from the Middle, 21*(2), 15-20.
Wilson, M. (2018). Making measurement important for education: The crucial role of classroom assessment. *Educational Measurement Issues and Practice, 37*(1), 5-20.
Zaslavsky, O. (2008). Meeting the challenges of mathematics teacher education though design and use of tasks that facilitate learning. In B. Jaworski & T. Woods (Eds.), *The mathematics teacher educator as a developing as a developing professional* (pp. 93-114). Rotterdam: Sense Publishers.
Zevenbergen, R. (2003). Grouping by ability: A self-fulfilling prophecy. *Australian Mathematics Teacher, 59*(4), 2-7.